Statistics 339: Lecture Topic 1
Introduction to Probability

Probability

- Probability (P) is a numerical measure of the likelihood that an event will occur.
- Probability values are always assigned on a scale from 0 to 1.
  - A probability near 0 indicates an event is unlikely to occur
  - A probability near 1 indicates an event is almost certain to occur

An Experiment and Its Sample Space

- An experiment is any process that generates well-defined outcomes.
- The sample space (S) for an experiment is the set of all experimental outcomes.
- A sample point (s) is an element of the sample space, any one particular experimental outcome.

Counting Rule for Multiple-Step Experiments

- If an experiment can be described as a sequence of k steps with n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by (n_1), (n_2)….. (n_k).

Counting Rule for Combinations

Another counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a a set of N objects.

Counting Rule for Permutations

A third counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects where the order of selection is important.

Assigning Probabilities

- Basic Requirements for Assigning Probabilities
  - The probability assigned to each experimental outcome must be between 0 and 1
  - The sum of the probabilities for all the experimental outcomes must equal 1.0
- Classical Method: Assigning probabilities based on the assumption of ***equally likely outcomes***.
- Relative Frequency Method: Assigning probabilities based on ***experimentation or historical data.***
- Subjective Method: Assigning probabilities based on the ***assignor's judgment.***

Classical Method

If an experiment has n possible outcomes, this method would assign a probability of $1/n$ to each outcome.

Experiment: Rolling a die

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Probabilities: Each sample point has a 1/6 chance

Relative Frequency Method

The probability assignments are given by dividing the number-of-days frequencies by the total frequency (total number of days).

Subjective Method

- When circumstances change rapidly it might be inappropriate to assign probabilities based solely on historical data, or perhaps little relevant data is available.

- We can use any data available as well as our experience and intuition, but a probability value should express our ***degree of belief*** that the experimental outcome will occur.
- Estimates often are obtained by combining the estimates from the classical or relative frequency approach with the subjective estimates.

Events and Their Probability

- An event (E) is a collection of sample points, that is, of possible outcomes from the experiment.
- The probability of any event is equal to the sum of the probabilities of the sample points in the event.
- If we can identify all the sample points of an experiment and assign a probability to each, we can compute the probability of an event.

Basic Concepts of Set Theory

- Set = a collection of objects which are called the members or elements of that set.
- Set Membership – if we have a set we say that some objects *belong* (or *do not belong*) to this set.

  $X \in A$ is read as x *belongs to* the set.
- Subset (C) – A_1 is a subset of A_2 *iff* every element in A_1 belongs to A_2
- Equality (=) - A_1 is said to be equal to A_2 *iff* every element in A_1 belongs to A_2 and every element in A_2 belongs to A_1.

4.3 Basic Relationships of Probability

Complement of an Event

- The complement of event *A* is defined to be the event consisting of all sample points that are not in *A*. (p. 155)
- The complement of *A* is denoted by A^c (p.155)

Union of Two Events

- The <u>union</u> of events *A* and *B* is the event containing all the sample points that are in *A* or *B* or both. (p.156)
- The union is denoted by $A \cup B$. (p.156)
- The number of elements in $A \cup B$ is either greater than or equal to the number of elements in either A or B.

Intersection of Two Events

- The <u>intersection</u> of events *A* and *B* is the set of all sample points that are in both *A* and *B.* (p.157)
- The intersection is denoted by $A \cap B$. (p.157)
- Note that if A_1 C A_2 then $A_1 \cap A_2$

Addition Law

- The <u>addition law</u> provides a way to compute the probability that event *A* or *B* or both occur. (p.157)
- The law is written as: $P(A \cup B) = P(A) + P(B) – P(A \cap B)$ p.157

Mutually Exclusive Events

- Two events are said to be mutually exclusive if the events have no sample points in common. (p.159)
- Two events are mutually exclusive if, when one event occurs, the other cannot occur. (p.159)
- We sometimes say that the events are disjoint.

Addition Law for Mutually Exclusive Events

$P(A \cup B) = P(A) + P(B)$

Partition

- A set of events $A_1, A_2, A_3 \ldots\ldots A_n$ partitions the sample space if the events are mutually exclusive (that is $A_i \cap A_j = 0$) and together they cover all of the sample space (S).
- The two conditions required for events to be a partition:
  - Their union is the sample space (S)
  - They are disjoint
- If either condition is violated, then the events do not partition the sample space.
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4.3 Conditional Probability

- The probability of an event given that another event has occurred is called a conditional probability. (p.161)
- The conditional probability of *A* given *B* is denoted by $P(A \mid B)$. (p.162)

Independent Events

- Two events A and B are independent if

$$P(A \mid B) = P(A)$$

Or

$$P(B \mid A) = P(B)$$

- If the probability of event A is not changed by the existence of event B we would say that events A and B are independent events. (p. 165)

Multiplication Law

- The multiplication law provides a way to compute the probability of an intersection of two events. (p. 165)
- The multiplication law is written as:

$$P(A \cap B) = P(B)P(A \mid B)$$

- The multiplication law for independent events

$$P(A \cap B) = P(A)P(B)$$

4.5 Bayes' Theorem

- Often we begin probability analysis with initial or prior probabilities.
- Then, from a sample, special report, or a product test we obtain some additional information
- Given this new information, we calculate revised probabilities referred to as posterior probabilities.

  (p. 169)
- Bayes' Theorem provides the means for revising the prior probabilities.